RATTLE VIBRATIONS IN MODELS OF GEARS

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Abstract

In this paper based on a gearbox model a suitable simplified model of the gear was built and analysed. The main focus of interest in this study was to determine the basic parameters affecting the gear rattle generation. The problem of meshing gear wheels with spur teeth which can be in mesh on the second line of action with taking into account random deviations of base pitches were taken into consideration. The deterministic model of the gear for idle running was examined by a given form of viscous damping. The type of vibrations was specified for different parameters and their levels as well as the search for the symptoms of chaotic motion for low levels of damping were also within our scope of interest. For the suitable combination of external forces, backlash, varying meshing stiffness as well as low damping, it appears that the contact of the driving flank of the tooth of the pinion and the opposite flank to the driven one of the tooth of the driven gear takes place.

Keywords: *transport, gears, vibration, rattle, damping*

1. Introduction

At certain conditions in gears a special type of vibrations occurs and it is caused by the existence of clearance. It commonly is known as rattle, hammering or clatter. Generation of these vibrations in gears is due to separation of the teeth on a theoretically correct line of action. In the case of instantaneous tooth engagements (so called vibro -impacts) these vibrations can appear on the second line of action, which corresponds to the reversible motion in relation to the nominal rotational motion of the gear. This phenomenon increases levels of vibrations and noise, especially at unloaded and lightly loaded gear transmissions e.g. in vehicles and machine tools.

The problems on rattle vibrations and noise is widely represented in the literature. Design practice in the field of building gearboxes proves that there is a manner of avoidance of rattle vibrations by means of experimental choice of parameters concerning elements of the whole driving system, and especially, the clutch. Singh, Xie and Comparin in [8] analysed from this aspect the driving system with a gear transmission and frictional dual-stage clutch having non-linear characteristic with hysteresis. The authors' goals was to examine the model of an automotive transmission, in which the rattle vibration problem by idle run is possible, and to verify existing formulae as well as build new ones in order to eliminate such vibrations by means of proper design and choosing parameters of the clutch. The computational problems, with regard to gear rattle for idle run, were also investigated by Padmanabhan et al [6]. The continuation and development of these subjects were Singh's and co-authors' papers (Kim and Singh [5]).

2. The Models of Gears for Examination of Rattle Problems

In the mathematical models in Singh's and his co-workers' papers [8], [5] there are introduced the drag torques, which are tied with each i gear wheel in the gear stage and described in the form of sums of components which are dependent of the rotational speed Ω_i and vibration velocity $\dot{\varphi}_{iv}$.

It is reasonable to examine the dynamic behaviour of the model shown in Fig.1, but in the case of damping forces described by the proportional mesh damping. The description of the variable gear meshing stiffness for spur gears and of deviations relevant to corresponding flanks of gear teeth is also introduced into the equations describing this model:

$$
\begin{aligned}
&I_1\frac{d^2\varphi_1}{dt^2}+T_{cv}\left(\frac{d\varphi_1}{dt},\frac{d\varphi_2}{dt},\varphi_1,\varphi_2,c_{c1},c_{c2}\right)+T_c\left(\varphi_1,\varphi_2,k_{c1},k_{c2},H_c\right)=T_e(t)\\
&I_2\frac{d^2\varphi_2}{dt^2}-T_{cv}\left(\frac{d\varphi_1}{dt},\frac{d\varphi_2}{dt},\varphi_1,\varphi_2,c_{c1},c_{c2}\right)+c_s\left(\frac{d\varphi_2}{dt}-\frac{d\varphi_3}{dt}\right)-T_c\left(\varphi_1,\varphi_2,k_{c1},k_{c2},H_c\right)+k_s(\varphi_2-\varphi_3)=0\\
&I_3\frac{d^2\varphi_3}{dt^2}-c_s\left(\frac{d\varphi_2}{dt}-\frac{d\varphi_3}{dt}\right)+c_g r_3\left(r_3\frac{d\varphi_3}{dt}-r_4\frac{d\varphi_4}{dt}\right)-k_s(\varphi_2-\varphi_3)+k_g(t)r_3F\left(t,r_3\varphi_3-r_4\varphi_4,x_{\mathrm{er}},x_\eta\right)=0\\
&I_4\frac{d^2\varphi_4}{dt^2}-c_g r_4\left(r_3\frac{d\varphi_3}{dt}-r_4\frac{d\varphi_4}{dt}\right)-k_g(t)r_4F\left(t,r_3\varphi_3-r_4\varphi_4,x_{\mathrm{er}},x_\eta\right)=0\,.
\end{aligned}
\tag{1}
$$

We assume next that in the system there is a dual stage clutch (dry friction clutch) with non-linear characteristics without hysteresis of the second stage. The formulae characterizing its stiffness and damping may be written in the following form:

$$
T_{cv}=\begin{cases}
c_{c1}\left(\dfrac{d\varphi_1}{dt}-\dfrac{d\varphi_2}{dt}\right) & \text{for}\quad |\varphi_1-\varphi_2|\le\Psi_{\lim}\\
c_{c2}\left(\dfrac{d\varphi_1}{dt}-\dfrac{d\varphi_2}{dt}\right) & \text{for}\quad |\varphi_1-\varphi_2|>\Psi_{\lim}
\end{cases}
$$

$$
T_c=\begin{cases}
k_{c1}(\varphi_1-\varphi_2)+\dfrac{H_c}{2}\operatorname{sgn}\left(\dfrac{d\varphi_1}{dt}-\dfrac{d\varphi_2}{dt}\right) & \text{for}\quad |\varphi_1-\varphi_2|\le\Psi_{\lim}\\
k_{c2}(\varphi_1-\varphi_2-\Psi_{\lim})+k_{c1}\Psi_{\lim} & \text{for}\quad \varphi_1-\varphi_2>\Psi_{\lim}\\
k_{c2}(\varphi_1-\varphi_2+\Psi_{\lim})-k_{c1}\Psi_{\lim} & \text{for}\quad \varphi_1-\varphi_2<-\Psi_{\lim}
\end{cases}\ , \tag{2}
$$

where: $\Psi_{\lim}$- transition angle of the clutch, H_c- hysteresis, sgn - sign for transfer function.
In the new co-ordinates ($\psi_i=\varphi_i-\varphi_{i+1}$; $i=1,2$; $\psi_3=\varphi_3-u\varphi_4$; u- gear ratio) after the elimination of rotational motion Eqs. (1) have the form:

$$
\begin{aligned}
&J_1\frac{d^2\psi_1}{dt^2}+T_{cv}\left(\frac{d\psi_1}{dt},\psi_1,c_{c1},c_{c2}\right)+T_c\left(\psi_1,k_{c1},k_{c2},H_c\right)-c_s\frac{I_1}{I_1+I_2}\frac{d\psi_2}{dt}-k_s\frac{I_1}{I_1+I_2}\psi_2=\frac{I_2}{I_1+I_2}T_e(t)\\
&J_2\frac{d^2\psi_2}{dt^2}+c_s\frac{d\psi_2}{dt}-\frac{I_3}{I_2+I_3}T_{cv}\left(\frac{d\psi_1}{dt},\psi_1,c_{c1},c_{c2}\right)-\frac{I_3}{I_2+I_3}T_c\left(\psi_1,k_{c1},k_{c2},H_c\right)+k_s\psi_2+\\
&\qquad\qquad -c_g r_3^2\frac{I_2}{I_2+I_3}\frac{d\psi_3}{dt}-k_g(t)r_3^2\frac{I_2}{I_2+I_3}F\left(t,\psi_3,\psi_{\mathrm{er}},\psi_\eta\right)=0\\
&J_3\frac{d^2\psi_3}{dt^2}-c_s\frac{I_4}{I_3u^2+I_4}\frac{d\psi_2}{dt}+c_g r_3^2\frac{d\psi_3}{dt}-k_s\frac{I_4}{I_3u^2+I_4}\psi_2+k_g(t)r_3^2F\left(t,\psi_3,\psi_{\mathrm{er}},\psi_\eta\right)=0
\end{aligned}\ , \tag{3}
$$

where: J_i - equivalent moment of inertia (i = 1,2,3); I_i- moments of inertia of the flywheel, the clutch hub, the input gear (equivalent value) and the output gear respectively, (cf Fig.1, i = 1,2,3,4); $r_3 = r_{b3}$, $r_4 = r_{b4}$ - base radii; c_{c1}, c_{c2}, c_s, c_g - viscous damping in clutch, in shaft, and equivalent value in the mesh zone; k_{c1},k_{c2}, k_s - stiffness by analogy with the foregoing; $k_g(t)$- global meshing stiffness- the value calculated for each zone of the mesh; F- symbolic description of the mesh function which depends on time, on the mesh zone, on static deflections, on current difference between values of single pitch deviations for the pinion and wheel ψ_{er} as well as on backlash ψ_η (in Eqs. (1) values x_{er} and x_η respectively). In calculation value of I_3 should include all the gear wheels that are in reality behind I_3 (moments of inertia I_5 up to I_8, except for the value I_4, remaining unchanged, r_5 up to r_8 - base radii respectively). Thus, for three gear pairs this moment of inertia is $I_3' = I_3+I_5+I_7+I_6(r_5/r_6)^2+I_8(r_7/r_8)^2$. Now one can observe the vibrations of the input gear with the moment of inertia considerably bigger than the one that describes the output wheel.

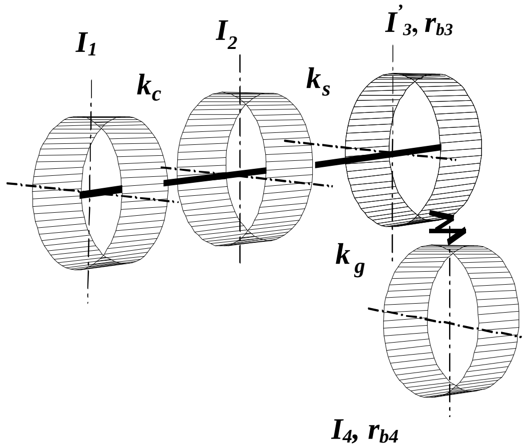


Fig.1. Simplified model of the gearbox [8]

Input torque excitation [6] has the form:

$$T_e(t) = T_{e0} + \sum_{l=1}^{n} T_l \sin[(N_e/2)l\Omega_e t + \varphi_{al}], \tag{4}$$

where: N_e -number of cylinders; l=1,2,...; Ω_e- engine rotational speed; φ_{al} -phase. By simulation of the idle run there are taken for simplicity only the first two engine harmonics and the mean engine excitation torque may be assumed as equal to zero.

Excluding deviations relevant to corresponding flanks of the gear teeth, the mesh function for constant stiffness may be written in the formula (in translational co-ordinates):

$$F(x,x_\eta) = \begin{cases} x, & x > 0 \\ 0, & -x_\eta \le \quad x \le 0, \\ x + x_\eta, & x < -x_\eta \end{cases} \tag{5}$$

where: x-relative displacements between gears (for a given stage of the gear $x = r_i\varphi_i - r_{i+1}\varphi_{i+1}$).

The presented considerations show the need for working out a simplified model of the gear. With regard to common usage of a single-degree- of- freedom model describing torsional vibration of gears, it is reasonable to examine its dynamic behaviour at the same conditions as the model described by Eqs. (3). It is obvious in such a case that we exclude possible influence of other nonlinearities in the whole system on dynamics except for the backlash in gears and that only basic physical parameters of the system (equivalent values) as well as of the gear wheels are within our scope of interest.

The introduction of dissipative forces in form of an equivalent viscous damping and variable stiffness for spur gears leads to the equation describing displacements of the gear that can be written as follows:

$$\frac{d^2x}{dt^2}+\frac{c_g}{M_Z}\frac{dx}{dt}+\frac{k_{gI}}{M_Z}\bar{f}(t,x,x_{er},x_\eta)=\frac{P}{M_Z}+\frac{T_1}{r_1}\frac{1}{m_1}\sin(2\Omega_e t)+\frac{T_2}{r_1}\frac{1}{m_1}\sin(4\Omega_e t)+\ldots, \quad (6)$$

where: M_z - equivalent mass reduced on the input shaft; m_1, m_2 - reduced mass of the pinion or of the wheel respectively; c_g - equivalent damping; k_{gI} - meshing stiffness of the single pair of teeth (generally, for low loads its value is less than the one for the nominal conditions); $\bar{f}(t,x,x_{er},x_\eta)$ - function describing the mesh and the changes of global meshing stiffness dependent of time, displacements, deviations of teeth and of the backlash; P - constant meshing force as a result of the static balance of the input and output torques; T_i - amplitudes of external input forces expressed by torques; $r_1 = r_{b3}$ - base radius of the pinion.

In the case of spur gears the static deflection x_{st} is determined by the use of the stiffness of the single-pair teeth. Now displacements are expressed in relation to the static deflection and to the nominal conditions of loads and ζ stands for the dimensionless damping factor. Introducing dimensionless relative mesh frequency $\nu = z_1\Omega_e/\omega_n$, and substituting dimensionless time $t^* = \omega_n t$ for real time t into Eq. (6), we can presented this equation of motion in the following form:

$$\frac{d^2z}{dt^{*2}}+2\zeta\frac{dz}{dt^*}+\bar{f}(t^*,z,\xi_{er},\eta)=B_0+B_1\sin\left(\frac{2\nu}{z_1}t^*\right)+B_2\sin\left(\frac{4\nu}{z_1}t^*\right)\ldots . \quad (7)$$

$B_0 = 0$ enables investigation of the gear motion in case of zero value term describing constant loads. Omitting the tooth deviations additionally leads to the theoretical description of a gear manufactured without errors.

In this case, the simplified description of meshing stiffness as by Sato et al.[7] was implemented and its values were referred to a single-pair of teeth. For cyclic time τ= t-entier(t/T_z)T_z and $\tau_1=0.1(\varepsilon_\alpha-1)T_z$ or $\tau_2=0.9T_z$ the dimensionless resultant stiffness is given in the form:

$$\begin{aligned}
&\text{for}\quad 0\le\tau<\tau_1 \rightarrow && k_g(t)=6\frac{2}{3}\frac{\tau}{(\varepsilon_\alpha-1)T_z}+1,\\
&\text{for}\quad \tau_1\le\tau<\tau_2 \rightarrow && k_g(t)=1\frac{2}{3},\\
&\text{for}\quad \tau_2\le\tau<T_z(\varepsilon-1) \rightarrow && k_g(t)=-6\frac{2}{3}\frac{\tau}{(\varepsilon_\alpha-1)T_z}+7\frac{2}{3},\\
&\text{for}\quad T_z(\varepsilon_\alpha-1)\le\tau<T_z \rightarrow && k_g(t)=1, \qquad (8)
\end{aligned}$$

where:
ε_α -tooth contact ratio.

The description of tooth deviations, which significantly influence the gear dynamics, was adopted from literature (e.g. from paper [2]). The case of single pitch deviations (pitch errors) for the pinion and the wheel is taken into account in the analysis. Suitable flank deviations having an influence on the gear dynamics are included in ISO [4].

3. Numerical Examples

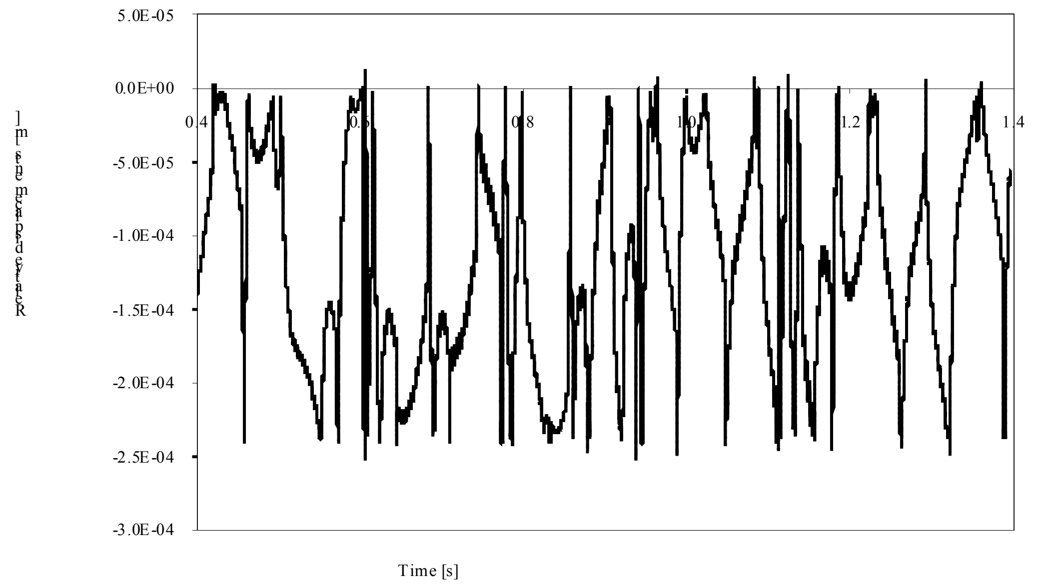
The computer simulation was based on the Eqs. (3) and (7). These equations were solved by the use of Gear's method. Computations were conducted for the gear with the number of teeth equal to z_1= 20 up to 24, z_2 =50 up to 57, for the tool displacement factors for the pinion and the toothed wheel x_1 in a range of 0 - 0.28 and x_2= 0, respectively and for module pitch of the gear m_n=2 mm.

According to the Eqs. (2), the non-linear characteristics of the clutch were introduced (k_c = 5 up to 10 Nm/rad and H_c = 0.15 Nm). The Fig.2a shows an example of relative displacements between gears for this model. In this case no teeth deviations was assumed (both toothed wheels were manufactured with no errors). It was also assumed, that the stage of the spur gear being analysed has the backlash within the boundaries x_η =0.08 - 0.24 mm - the rest of data is based on the analysis from the paper [8].

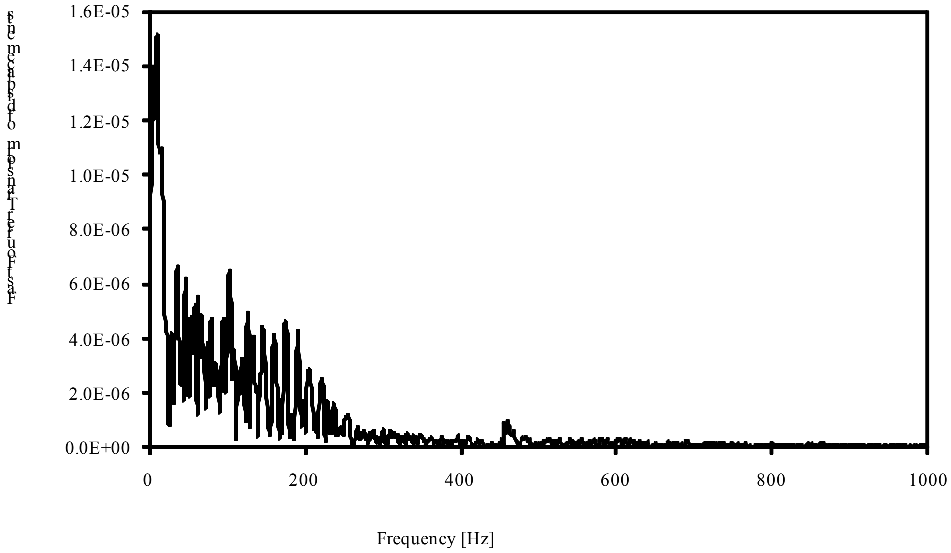
The vibration spectrum (Fig.2b - Fast Fourier Transform - FFT method) depends on the damping level, which is determined by the use of an equivalent system describing the same gear and groups its significant components at the lowest band of frequencies. It consists of the fundamental engine harmonics compatible with the force spectrum. The first (cir. 10 Hz) and second natural frequency (cir. 470 Hz) is clearly visible too. It is easy to notice, that at the spectrum no frequencies are connected with the mesh frequency f_Z =360 Hz.

Reducing Eqs. (3) in order to examine only the description of the gear (Eq.(7)), the simulation was performed for the following cases:

1. For the equivalent moment of inertia for all moments connected with the input shaft and the whole system considered in Eqs. (7),
2. With regard to all the moments of inertia which there are behind the considered gear and are connected with the input shaft ($I_1=I_3'$),
3. For the isolated gear, with only actual parameters I_1 and I_2.



(a)

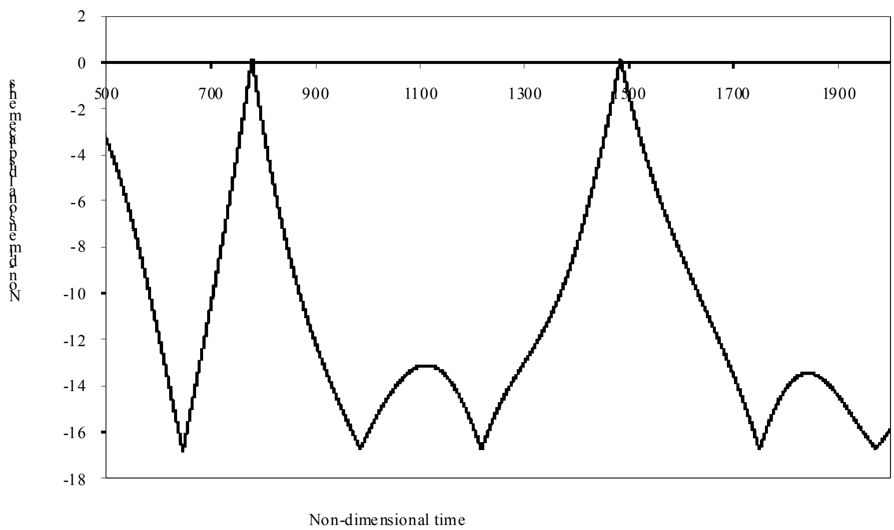


(b)

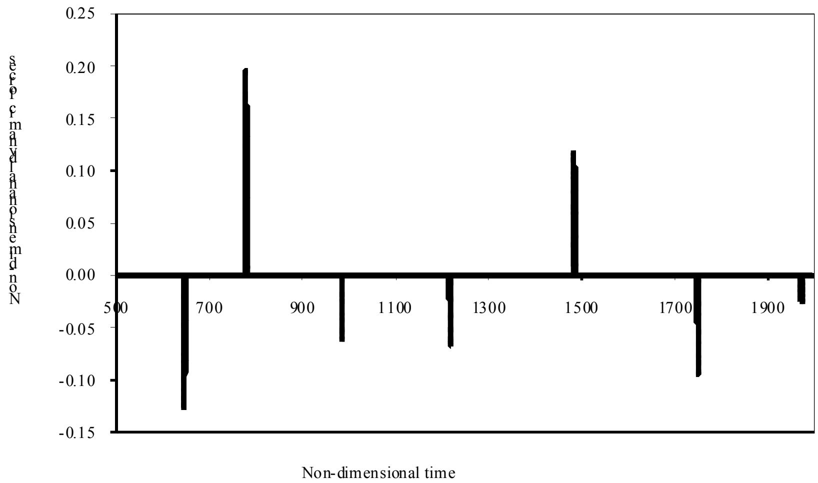
Fig.2. Examples of relative displacements between gears (a) and theirs spectrum (b) for Eqs (3)

The first terms of the time series (Eq. (4)) were accepted as the relation $T_1/T_N=(T_1/r_1)/P_N=0.25932$ and $T_2=T_1/4$. The results are presented in Fig. 3. As it is visible on the basis of examples shown here, for the constant level of damping the double-sided rattle vibration imply generation of considerable dynamic forces. In the case of data for an isolated gear it gives unrealistic values of frequencies and amplitudes. For this reason, it is necessary to introduce into the Eq. (7) the equivalent parameters of mass and inertia (the first and second versions of data).

The example illustrated in the Fig.4, refers to the gear with random base pitch deviations $|x_{pi}|<0.5$ and $|x_{pj}|<0.6$ (ranges of values of the base pitch deviations x_{pi}, x_{pj} for both toothed wheels in relation to the static deflection $i=1, ..., z_1$; $j=1,..., z_2$). The spectrum appropriate for this case is presented in the Fig.4b. In the last case the abscissa is described by the relative frequency f*.



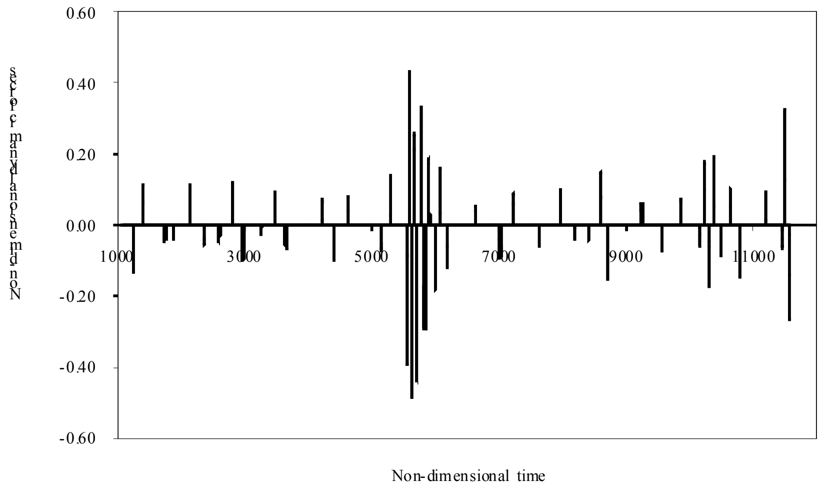
(a)



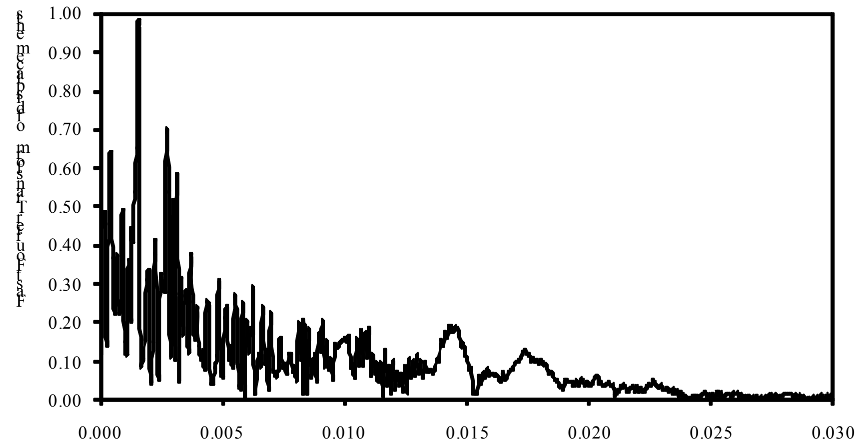
(b)

Figs.3. Relative displacements between gears (a) and dynamic forces (b) for the constant damping level $\zeta=10^{-3}$ and versions of data (2), respectively

The dimensionless damping factor was equal to: 10^{-6}, 10^{-5}, 10^{-4}, 10^{-3} or 10^{-2}. In the last case we obtain steady periodic vibrations with two predominant frequencies that correspond to the fundamental frequencies of the external excitation. So, in the spectrum we can observe relative frequencies corresponding with the real engine frequencies $f_{01}=\Omega_e/\pi$ and $f_{02}=2\Omega_e/\pi$, as well as the rotational frequency $f_1=\Omega_e/2\pi$ and its harmonics. Similarly, as in the previous examined model, at the spectrum there are not any frequencies connected with the mesh frequency (in this case it would be a spectral line for the dimensionless relative frequency $f^*=0.01859245$). For example for $\zeta=10^{-3}$ the most distinctive is the frequency $f^*= 0.00155$ that with a high degree of accuracy, corresponds to $f_{01}= 15$Hz, while the other important frequencies correspond with its harmonics. It is worth mentioning that for the considered variants of data, the value of the natural frequency f_3 is very close to that calculated from Eq. (7). However, it is true only under the condition that inertial parameters are determined for the equivalent system with input moments of inertia referring to the clutch shaft. By the diminished level of damping the vibration spectrum broadens and simultaneously the band of important frequencies is 'condensed'.



(a)



(b)

Fig.4. Dynamic forces (a) and Fourier transform for the displacements (b), $\zeta = 10^{-3}$, $\eta = 16.6666$, $|x_{pi}| < 0.5$ and $|x_{pj}| < 0.6$

The research into possibilities of generation of the double-sided impacts and the ranges of values of parameters giving such a type of vibration can be effectively using bifurcation diagrams. For this purpose the results of simulations were sampled according to the meshing period. In this way we obtained diagrams that were a source of information on the vibration type as well as the limits of the displacements in gears (cf. Dyk [3]). The example is presented in Fig. 5 for the results sampled according to the mesh period made on the basis of the calculations for the constant level of damping and varying backlash.

The problem of chaotic motion in the models of gears was researched, among others, in the papers mentioned in section 1. In the paper [7] (Sato et al.) Lyapunov exponents were calculated for the similar form of equations. The considered solutions of equations without random excitation for low levels of the viscous damping also reveal positive values of Lyapunov exponents (exponents on the basis of a time history cf [1]). For instance for the data as in the example shown in Fig. 4 but without the base pitch deviations x_{pi}, x_{pj} , according to the Eqs.(7) and (8), the maximal exponent is equal to 0.0097319731; enlarging the damping up to $\zeta = 10^{-2}$ gives the negative values practically near zero.

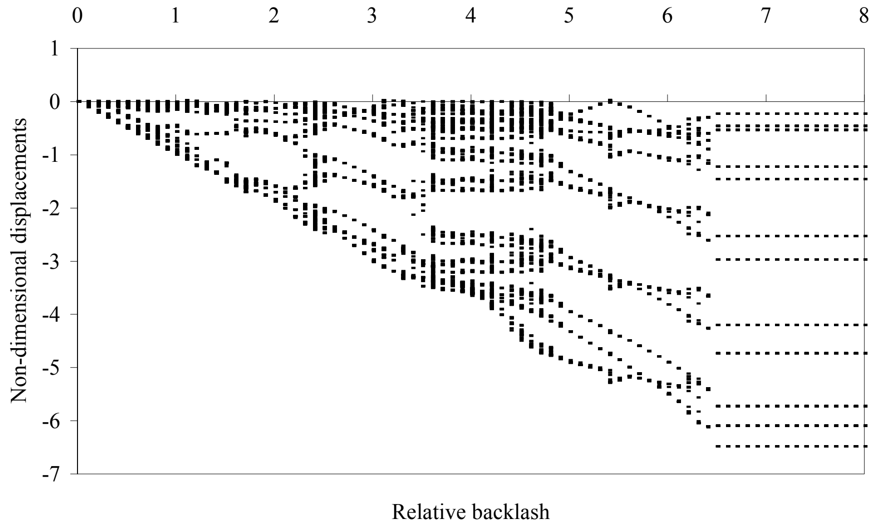


Fig.5. Bifurcation diagram (z-η plane, constant value of dimensionless damping factor $\zeta = 0.01$)

4. Concluding Remarks

The rattle vibrations develop in the low damped system, which is subjected to the variable external load with the zero (or very low) average components. The determination of the range of parameters, which affect the generation of the rattle vibration, can be useful in attempts to eliminate it from real systems.

The most frequent is the lack of contact of the driving and driven flanks of teeth. It results in so-called single-sided impacts. For the suitable combination of external forces, backlash, varying meshing stiffness as well as low damping, it appears that the contact of the driving flank of the tooth of the pinion and the opposite flank to the driven one of the tooth of the driven gear takes place (double-sided impacts).

It is worth mentioning, that by the low damping levels when for considered descriptions and amount of the external excitation, often the aperiodic, chaotic (and random, for Gaussian distribution of the pitch deviations) character of vibration is manifested. For this reason, it is impossible to predict precisely the dynamic loads.

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